SIFRE: Simulation of Flexible and Renewable Energy sources

Abstract

This paper introduces the market simulation tool SIFRE - SImulation of Flexible and Renewable Energy sources. SIFRE is based on the Unit Commitment problem but includes much greater detail on fuel consumption, on multiple energy types and on connected energy systems. It allows for circles in the system, for example in the case of electrolysis where electricity is converted to gas, which can be converted back to electricity. The goal of SIFRE is to support highly flexible and integrated energy systems in great detail and in reasonable simulation time, such that the future behavior of energy system can be analyzed. The offset is the Danish heat and power system and SIFRE supports wind power and Combined Heat and Power generation in great detail. The tool is, however, not hardcoded to any energy system and can thus be applied however liked. SIFRE complements existing market simulation tools with its high level of detail, flexibility and support of integration of multiple energy systems. Backtest on historical data indicates that SIFRE is capable of producing high quality results within reasonable time.

Content

1. Introduction

In this paper, a new energy market simulation tool is introduced: SIFRE – Simulation of Flexible and Renewable Energy sources. The goal of SIFRE is to simulate the spot market behavior for energy systems and to facilitate analyses of highly flexible and integrated systems.

SIFRE is based on a MILP formulation for the Unit Commitment (UC) problem. The UC problem provides an optimal production schedule such that energy demand is met. In the literature, UC formulations consider the production of energy without taking into account details about fuel consumption. Instead, fuel consumption is represented by a (non-linear) production cost. The UC problem has been solved using Lagrangian relaxation and dynamic programming. More recently, however, MILP solvers have become so powerful that solving the UC as a MILP is an attractive alternative. See [1] [2] [3] [4] for literature surveys of the UC problem.

When simulating a flexible and integrated energy market, the UC formulation should not be restricted to one energy type but instead facilitate the integration of several energies. Also, it is not sufficient to only consider the production side of generators; fuel consumption must be represented in detail as fuel could be produced by another generator (heat pumps convert electricity into heat, electrolysis converts electricity into gas, etc.). Fuel consumption is an important result when simulating and analyzing the behavior of energy systems. In the remainder of this paper, UC formulation refers to such an extended formulation, unless else noted.

The offset of SIFRE is the Danish heat and power systems, which are closely coupled through significant amounts of installed capacity at Combined Heat and Power production plants (CHP). The Danish system also holds a large amount of renewable energy, which brings along the need for analyzing flexibility. The proposed mathematical formulation is capable of handling renewable energy and the desired flexibility. The formulation is generic and not hardcoded to any specific energy system (not even the Danish): data and parameters for each component in the energy system are input. SIFRE aims at facilitating a very flexible and generic representation of the energy system, without restricting which energies to produce and which to consume. SIFRE is thus a generic simulation tool and is not focused on a specific energy system or geographic area. Hydro power, including pump storages, is currently not supported in SIFRE, but other than that the generic design of SIFRE facilitates modelling of power markets, district heating, gas, transportation, etc. either as several closed systems or in a single integrated energy system.

2. Design of the energy system

In SIFRE, an energy system is represented using the overall building blocks: areas, conversion units, storages and interconnection lines.

- Areas represent a geographical area and an energy type. Examples are district heating in Copenhagen; electricity in SE1; coal in Poland. Energy consumptions are attached to an area, for example the district heating consumption in Copenhagen; and the electricity consumption in SE1.

In case of fuels, the available amount and price of the fuel is also attached to the area, for example the available amount and price of coal in Poland.

- Production units convert energy. They connect areas via directed edges. Areas with edges **to** the conversion unit provide fuel and areas with edges **from** the conversion unit receive produced energy. An example is a heat boiler converting natural gas to district heating; in this case a gas area has an edge **to** the unit and a district heating area has an edge **from** the conversion unit.
- Storages are connected to areas. Only short-term storages are supported in the proposed formulation (i.e. hydro power is not modelled).
- Interconnection lines connect two areas, which consist of the same energy type (not just electricity areas).

With this in mind, an example of an energy system can be formed. Let **circles** represent areas, **squares** conversion units, **triangles** storages, and **bold edges** interconnection lines. The illustration in Figure 1 is an example. There is no restriction on the number of building blocks and there is no restriction on which energy types to include . As illustrated in the example, the user can introduce a system which transforms electricity into gas and in this way introduces cycles in the energy system.

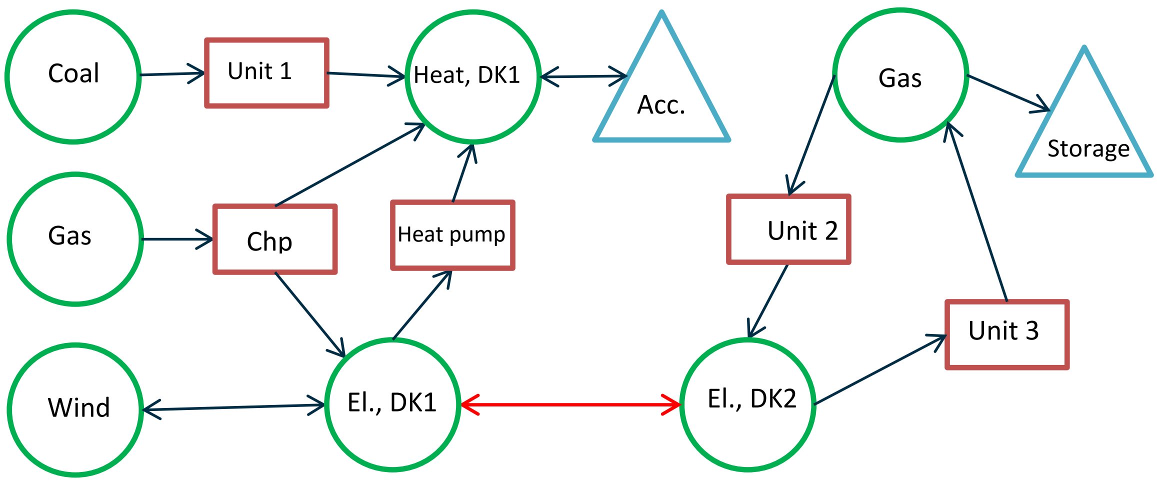

Figure 1 Example on how an energy system can be represented in SIFRE. The example only illustrates part of the available functionality. The number of areas, conversion units, storages, etc. is not limited.

Some overall design decisions must be satisfied when modelling an energy system:

- An interconnection line should only connect two areas of same energy type
- An unlimited number of areas can be connected *to* a production unit
- Production conversion unit can produce energy to at most two areas

3. Simulation of the energy system

The goal of SIFRE is to simulate the spot market. To do this, the tool solves a mathematical problem with binary UC variables. The UC variables are used to include startup costs, which again are used to represent that generation plants in the real world often submit *block bids*, i.e., bids spanning several hours or even days. Including startup costs will ensure that generation units are turned on in longer periods of time.

Generation and consumption bids are submitted to the spot market by stakeholders, who use the current knowledge and predicted behavior of the system to decide their bids. The predictions can only be assumed to be somewhat accurate for the next brief time period, e.g. for the next week. SIFRE thus simulates the spot market one week at a time. To prevent that all generation units turns off and storages are emptied at the end of the week, SIFRE simulates nine days but only uses the results from the first seven days.

Seasonal and long-term storages are optimized to utilize the expected future behavior. It is thus not sufficient to only consider the next week, when determining the desired inventory levels for seasonal and long-term storages. SIFRE supports such storages by solving the LP-relaxed UC problem for a full year in hourly time resolution. The resulting storage levels for the end of each week are then transferred to the weekly (9 days) simulations described above. If the storage levels are not satisfied, a penalty must be paid in the weekly (9 days) simulations. The penalty reflects the highest energy price, the energy could have been sold for in any later week.

Maintenance schedules on generation plants should be taken into control. In real life, maintenance schedules are typically coordinated across production companies to prevent power shortage. SIFRE simulates this by solving the UC problem including maintenance requirements for a full year with low detail (e.g. a time step corresponds to a day or a week). The resulting maintenance plan is used in the detailed (hourly) simulation

The SIFRE algorithm, which thus consists of two layers:

- Layer 0 simulates a full year with hourly time resolution, where the UC problem is LP-relaxed, such that the problem is solvable in reasonable time. The purpose of layer 0 is to decide the weekly desired storage levels and the penalty for not satisfying them. The result of layer 0 is transferred to layer 1 and 2.
- Layer 1 simulates a full year with low detail (a time step consists of one day or one week). The purpose of layer 1 is to decide the timing for maintaining of conversion units.
- Layer 2 typically simulates 9 days at a time with full detail (a time step consists of one hour). The purpose of layer 2 is to simulate the spot market and the results from the first 7 days are saved. The remaining two days of the simulation period prevents that the system shuts down at day 7. Layer 2 uses information on maintenance and storage levels from layer 1. The maintenance periods must be satisfied in layer 2. The storage levels are used as indicates for the desired levels at the end of each simulation periods, such that all storage is not used immediately in the first simulation period.

4. Notation

Notation is needed to formulate the unit commitment problem mathematically. First the sets:

N	the set of areas
H	the set of conversion units
R	the set of energy storages
T	the set of time steps
$A^{ICL} \subseteq N \; x \; N$	the set of interconnection lines, each connecting two areas of same energy type
$A \subseteq H \; x \; N \cup R \; x \; N \cup R_E \; x \; N \cup R_E \; x \; E$	the set of arcs connecting conversion units and areas, and connecting storages and areas, incl. those for electric vehicles
$RAMP$	The set of all ICL ramping conditions
$A_m^{RAMP+} \subseteq A^{ICL}$, $A_m^{RAMP-} \subseteq A^{ICL}$	Subset of ICL for positive / negative ramping
D	the set of flexible consumptions of type “price cut”. $\ell \in D(i)$ gives a vector of flexible consumptions ℓ in area $i \in N$. The vector has the length of the time horizon and each element gives information on the amount of flexible consumption in the corresponding time step
F	the set of flexible consumptions of type “load shift”. $j \in F(i)$ gives a vector of flexible consumptions j in area $i \in N$. The vector has the length of the time horizon and each element gives information on the amount of flexible consumption in the corresponding time step
L	Steps for over and under production

Parameters are needed to represent data and to couple the different parts of the formulation:

$\hat{\psi}_i^t \in \boldsymbol{R}$	The inflow cost for area $i \in A$ in time step $t \in T$
$d_i^t \geq 0$	The sum of fixed consumption (demand) in area $i \in N$ and in time step $t \in T$
$co_i^l, cu_i^l \geq 0$	The cost for over- resp. under-production in area $i \in A$ in step $l \in L$
$c_{ih}^t \in \boldsymbol{R}$	The cost of using energy from area $i \in A$ in unit $h \in H$ in time step $t \in T$
$\ddot{c}_{hi}^t \in \boldsymbol{R}$	The cost of producing energy to area $i \in A$ by unit $h \in H$ in time step $t \in T$
$\bar{\bar{c}}_{jt}^{t'} \in \boldsymbol{R}$	The cost of shifting flexible demand $j \in F(i), i \in N$ from time step $t \in T$ to time step $t' \in T: t - k_j \leq t' \leq t + k_j$
$\tilde{c}_h^t \in \boldsymbol{R}$	The O&M cost for unit $h \in H$ in time step $t \in T$
$\bar{c}_{ij}^t \in \boldsymbol{R}$	The cost of using interconnection line $(i, j) \in A^{ICL}$ in time step $t \in T$
$0 \leq \gamma_{ij}^t \leq 1$	The loss for interconnection line $(i, j) \in A^{ICL}$ in time step $t \in T$
$x_{ij}^{START} \in \boldsymbol{R}$	The initial flow on interconnection line $(i, j) \in A^{ICL}$
$\hat{\rho}_r^{END} \in \boldsymbol{R}$	The profit of keeping energy on storage $r \in R$ at the end of the simulation period
$\pi_\ell^t \in \boldsymbol{R}$	The maximal energy price, before flexible energy $\ell \in D(i), i \in A$ is cut in time step $t \in T$
$0 \leq \gamma_{ri}^t \leq 1$	The loss when extracting from storage $r \in R$ to area $i \in N$ in time step $t \in T$
$0 \leq \gamma_{ir}^t \leq 1$	The loss when injecting to storage $r \in R$ from area $i \in N$ in time step $t \in T$
$0 \leq \gamma_r^t \leq 1$	The storage loss for storage $r \in R$ in time step $t \in T$
$s_r^{START} \geq 0$	The initial storage level for storage $r \in R$
$\bar{s}_r^t \in \boldsymbol{R}$	The maximum storage capacity for storage $r \in R$ in time step $t \in T$
$\underline{s}_r^t \in \boldsymbol{R}$	The minimum storage capacity for storage $r \in R$ in time step $t \in T$
$\bar{v}_{ir}^t \in \boldsymbol{R}$	The maximum injection rate from area $i \in N$ to storage $r \in R$ in time step $t \in T$
$\bar{v}_{ri}^t \in \boldsymbol{R}$	The maximum extraction rate from storage $r \in R$ to area $i \in N$ in time step $t \in T$
$\bar{x}_{ij}^t \geq 0$	The maximum capacity for flow travelling from area $i \in N$ to area $j \in N$ in time step $t \in T$
$\chi_{ij}^t \geq 0$	The ramping limit on interconnection line $(i, j) \in A^{ICL}$ in time step $t \in T$
$\bar{p}_{hi}^t \geq 0$	Technical production maximum for unit $h \in H$, when producing to area $i \in N$ in time step $t \in T$
$\underline{p}_{hi}^t \geq 0$	Technical production minimum for unit $h \in H$, when producing to area $i \in N$ in time step $t \in T$

$\mathrm{P}_{hi}^{t} \geq 0$	The ramping limit on production unit $h \in H$ for production to area $i \in N$ in time step $t \in T$
$c_v, c_b \geq 0$	Constants used in the PQ diagram for a CHP
$a, b, c \geq 0$	Constants used in the efficiency for a conversion unit
$0 \leq m_{ih} \leq 1$	Maximum usage in percent of fuel $i \in N$ at conversion unit $h \in H$ in time step $t \in T$
$0 \leq m_{ih}^{s} \leq 1$	Maximum usage in percent of startup fuel $i \in N$ at conversion unit $h \in H$ in time step $t \in T$
$k_{ih} \geq 0$	Correction factor for conversion unit $h \in H$ using fuel $i \in N$
$\ddot{a}, \ddot{b}, \ddot{T} \geq 0$	Constants used to derive the startup fuel consumption of a conversion unit. $\ddot{a}$ is an overall consumption, $\ddot{b}$ is a consumption, which depends on how long the unit has been offline. $\ddot{T}$ is a time constant used to weigh the offline time
$k_J \geq 0$	The amount of time, flexible demand $j \in F(i), i \in N$ can be shifted
$\bar{d}_{\ell}^{t} \geq 0$	The maximum amount of flexible demand $\ell \in D(i) \cup F(I)$ in time step $t \in T$
$\overline{\Psi}_{i}^{t} \geq 0$	The maximum inflow amount to area $i \in N$ in time step $t \in T$
$\underline{\Psi}_{i}^{t} \geq 0$	The minimum inflow amount to area $i \in N$ in time step $t \in T$
$M_h \geq 0$	Number of yearly maintenance periods for unit $h \in H$
$m_h \geq 0$	Maintenance time for unit $h \in H$
$\underline{m}_h, \overline{m}_h \geq 0$	Minimum and maximum time between two revision periods for conversion unit $h \in H$
$p_{out}^{h}, l_{out}^{h} \geq 0$	Percentage in outage and average outage length for unit $h \in H$
$0 \leq K_e \leq 1$	Percentage of electric vehicles unavailable to the power system grid at the highest value for electric vehicle consumption, for electric vehicles $e \in E$

Variables in the formulation are:

$\tilde{d}_{i\ell}^{t} \geq 0$	The flexible consumption (demand) $\ell \in D(i)$ in area $i \in N$ and in time step $t \in T$
$\ddot{d}_{ij}^{t\tau} \geq 0$	The flexible consumption (demand) $(j, t) \in F(i, t)$ in area $i \in N$, covered in time step $\tau \in T$
$p_{hi}^{t} \geq 0$	The production from conversion unit $h \in H$ to area $i \in N$ in time step $t \in T$
$f_{ih}^{t} \geq 0$	The fuel from area $i \in N$ used by conversion unit $h \in H$ in time step $t \in T$
$f_{ih}^{ts} \geq 0$	The startup fuel from area $i \in N$ used by conversion unit $h \in H$ in time step $t \in T$
$x_{ij}^{t} \in \boldsymbol{R}$	the amount of flow from area $i \in N$ to area $j \in N$ in time step $t \in T$
$v_{ir}^{t} \geq 0$	The amount of energy injected from area $i \in N$ to storage $r \in R$ in time step $t \in T$
$v_{ri}^{t} \geq 0$	The amount of energy extracted from storage $r \in R$ to area $i \in N$ in time step $t \in T$
$s_{r}^{t} \geq 0$	The storage level in storage $r \in R$ in time step $t \in T$
$o_{i}^{lt} \geq 0$	The amount of overproduction in area $i \in N$ in time step $t \in T$ for step $l \in L$
$u_{i}^{lt} \geq 0$	The amount of underproduction in area $i \in N$ in time step $t \in T$ for step $l \in L$
$\psi_{i}^{t} \geq 0$	The amount of fuel available in area $i \in N$ in time step $t \in T$ (inflow)
$z_{h}^{t} \in \{0, 1\}$	Denotes if conversion unit $h \in H$ is online or offline in time step $t \in T$
$y_{h}^{t\ell} \in \{0,1\}$	Denotes if conversion unit $h \in H$ is turned on or not in time step $t \in T$ after having been offline in ℓ timesteps
$zm_{h}^{t} \in \{0,1\}$	Denotes if unit $h \in H$ starts a maintenance period at time step $h \in H$

5. Problem definition

The mathematical formulation shares great similarities to the state of the art UC formulations [1] [3]. Constraints or functionality, which varies from existing formulations, are highlighted. It is noted that the mathematical formulations are not available for many of the commercial energy market simulators; hence the comparison to existing work is limited to published material.

The goal is to minimize the total costs:

$$\min Z = \sum_{t\in T}\left(\sum_{i\in N}\left(\widehat{\psi}_i^t\psi_i^t + \sum_{l\in L(i)}(co_i^l o_i^{lt} + cu_i^l u_i^{lt}) + \sum_{(i,h)\in A} c_{ih}^t(f_{ih}^t + f_{ih}^{ts}) + \sum_{(h,i)\in A} \dddot{c}_{hi}^t p_{hi}^t\right) + \sum_{(i,j)\in A^{ICL}} \vec{c}_{ij}^t x_{ij}^t + \sum_{h\in H} \tilde{c}_h^t z_h^t\right) - \sum_{r\in R} \hat{\rho}_r^{END} s_r^{t'} - \sum_{t\in T}\sum_{i\in N}\sum_{\ell\in D(i)} \pi_\ell^t \widetilde{d}_\ell^t + \sum_{i\in N}\sum_{(j,t)\in F(i)}\sum_{\tau=t-k_j}^{t+k_j} \bar{\bar{\mathrm{c}}}_{\mathrm{jt}}^{\tau} \ddot{d}_j^{t\tau} \tag{1}$$

The cost function consists of a number of components, which are described in greater detail in the following sections. The costs are:

- The inflow cost $\widehat{\psi}_i^t$ for area $i \in N$, time step $t \in T$. For fuel (or wind) areas, this corresponds to the cost of purchasing fuel externally (producing wind)
- The cost of over- and underproduction, co_i^l, cu_i^l for area $i \in N$, time step $t \in T$ in step $l \in L$. Over- and under-production occurs when market clearing is not possible: the value of over and underproduction should thus be set to high costs. The steps for over and underproduction are necessary to support spreading out over and underproduction. Assume that the costs increase with each step; then the objective function seeks to only activate the lower shifts and, in this way, will distribute over and underproduction across more areas if possible
- Fuel consumption costs c_{ih}^t for area $i \in N$, conversion unit $h \in H$, time step $t \in T$. Consists of fuel consumption costs such as emission costs, subsidies and taxes; both for regular fuel usage and for startup fuel usage. Fuel purchase costs are not included, because fuel is not necessarily purchased externally. Consider the case of electrolysis in the energy system, where gas can be generated by a conversion unit. The fuel cost for gas is then the production cost for the conversion unit and not some external, fixed fuel cost
- Production cost, $\dddot{c}_{hi}^t$, for conversion unit $h \in H$, area $i \in N$, time step $t \in T$. Consists of costs such as subsidies and taxes, and possibly emissions
- ICL costs, $\vec{c}_{ij}^t$, for flow travelling from area $i \in N$ to area $j \in N$ in time step $t \in T$. The value of the cost must be set as follows:
    - If the interconnection line is between two "internal" areas, i.e., areas where the area price is not yet determined, then the cost $\vec{c}_{ij}^t$ is set to the net tariff

- If the interconnection line represents export to external area $j \in N$, then the cost $\bar{c}_{ij}^{t}$ is set to the negated area price of external area $j \in N$, as the price represents the income for selling income to $j \in N$. Add to this the net tariff
- If the interconnection line represents import from external area $i \in N$, then the cost $\bar{c}_{ij}^{t}$ is set to the area price of external area $i \in N$. Add to this the net tariff

- Hourly operation costs, $\tilde{c}_{h}^{t}$, for conversion unit $h \in H$, time step $t \in T$
- Storage level indicator costs, $\hat{\rho}_{r}^{END}$, for storage $r \in R$. The cost prevents stored energy to be used too soon in the simulation. In layer 0 and 1, the storage is not emptied at the end of the year. In layer 2, the storage is not emptied in the first simulation period. In layer 2, the cost $\hat{\rho}_{r}^{END}$ is set using the area price from layer 1 for the area, to which the storage is attached. Specifically, storing energy at the end of a simulation period in layer 2 is rewarded by the area price from layer 0 for the following simulation period. If the area price is low in the following simulation period, then the stored energy also has low value and less energy should be stored. If the area price is high in the following simulation period, then it may be worthwhile to store more energy
- The maximum price of flexible demand, π_{ℓ}^{t}, for flexible demand of type price cut $\ell \in D(i)$, in area $i \in N$, time step $t \in T$. If the area price exceeds π_{ℓ}^{t}, the demand is dropped. Consider the objective function:
  - if the sum of costs for producing the flexible demand exceeds the profit, then the total objective function value increases. As we want to minimize costs, it would not be worthwhile to satisfy the demand
  - if the sum of costs for producing the flexible demand does not exceed the profit, then the demand is satisfied because this would yield a lower objective function value
- Cost $\bar{\bar{c}}_{jt}^{\tau}$ of shifting flexible demand $j \in F(i), i \in N$ from time step $t \in T$ to time step $\tau \in T: t - k_J \leq \tau \leq t + k_j$

As stated in the beginning of the section, the mathematical formulation minimizes cost, which corresponds to maximizing the social welfare. This, however, depends on the input to the model. If a simulation includes subsidies and tariffs, which do not stem from social welfare economics, then the result will also not represent the welfare economics. Taking this into account, SIFRE is capable of simulations both according to social welfare economics and to business economics.

5.1 Energy balance constraint

In each area, the amount of ingoing energy must equal the amount of outgoing energy incl. energy consumption:

$$\psi_i^t + \sum_{(h,i)\in A} p_{hi}^t - \sum_{(i,h)\in A} (f_{ih}^t + f_{ih}^{ts}) + \sum_{(j,i)\in A^{ICL}} x_{ji}^t - \sum_{(i,j)\in A^{ICL}} \gamma_{ij}^t x_{ij}^t - \sum_{(r,i)\in A} (v_{ir}^t - \gamma_{ri}^t v_{ri}^t) - o_i^t + u_i^t \quad (2)$$
$$= d_i^t + \sum_{\ell \in D(i)} \tilde{d}_{i\ell}^t + \sum_{t' \in F(i)} \ddot{d}_j^{t't}, \qquad \forall i \in N, t \in T$$

The constraint says that the demand in an area must be covered by:

- the sum of inflow,
- net production: production minus consumptions for this energy type,

- net import: import minus export,
- net extractions from storages: extraction minus injection,
- net overproduction: overproduction minus underproduction

The inflow and demand variables are described in more detail.

5.1.1 Inflow

Constraints (2) contain inflow variables representing the energy, which is injected into the area in each time step. If the area represents a fuel, e.g., coal, then the inflow corresponds to how much coal is purchased externally by the fuel area in each time step. If the area represents wind, then the inflow corresponds to how much wind energy is available for the wind area in each time step. Inflow can be bounded from above and below. The upper bound represents that the amount of energy is not necessarily unlimited. The lower bound represents that some energy *must* be used as is the case for e.g. wind without curtailment possibilities.

$$\underline{\Psi}_i^t \leq \psi_i^t \leq \overline{\Psi}_i^t, \qquad \forall i \in N, t \in T \tag{3}$$

In the literature, fuel consumption is handled implicitly, see e.g. [1] [2] [3] [4], because it is assumed that fuel is either purchased externally (e.g. coal, oil) or produced by units (electricity). SIFRE allows a mix of these; the cost of fuel thus depends on the source of the fuel and must kept separately from the fuel consumption cost. Including inflow variables sets SIFRE apart from the formulations in the literature.

5.1.2 Demand

The demand (or consumption) on the right-hand side of constraint (2) can be fixed or flexible. The fixed demand d_i^t must always be satisfied. Two types of flexible demand are supported by SIFRE:

- Load shift demand, which can be moved forwards or backwards in time if beneficial
- Price cut demand, which can be dropped if the energy price is above a given threshold

5.1.2.1 Load shift demand

Load shift demand is formulated as:

$$\sum_{\tau=t-k_j}^{t+k_j} \ddot{d}_j^{t\tau} = \overline{d_j^t}, \qquad \forall i \in N, (j,t) \in F(i) \tag{4}$$

The demand must be covered within its time frame.

5.1.2.2 Price cut demand

Price cut demand is included by a reward in the objective function: If the reward is greater than the total costs of satisfying the costs, then the demand is satisfied. The reward is thus the threshold, which decides when the demand is dropped.

The price cut demand is included in the balance constraint on the right-hand side as variable $\tilde{d}_{i\ell}^t$.

5.2 Storages

The storage level depends on the amount of injection and extraction:

$$s_r^t = (1-\gamma_r^t)s_r^{START} + \sum_{\substack{i \in N: \\ (i,r),(r,i) \in A}} \left((1-\gamma_{ir}^t)v_{ir}^t - v_{ri}^t\right), \quad \forall r \in R, t = 0 \tag{5}$$

$$s_r^t = (1-\gamma_r^t)s_r^{t-1} + \sum_{\substack{i \in N \\ (i,r),(r,i) \in A}} \left((1-\gamma_{ir}^t)v_{ir}^t - v_{ri}^t\right), \qquad \forall r \in R, t \in T\backslash\{0\} \tag{6}$$

The first constraints ensure that the storage level is correct after the initial time step while the second constraint ensures that the storage level is correct after any other time step. The constraints take into account the storage level in the last time step, the storage loss, injection to the storage including injection loss, and extraction from the storage.

Bounds are given for storage levels and for injection and extraction rates:

$$\underline{s}_r^t \le s_r^t \le \overline{s}_r^t, \qquad \forall r \in R, t \in T \tag{7}$$

$$v_{ir}^t \le \underline{v}_{ir}^t, \qquad \forall r \in R, i \in N: (i,r) \in A, t \in T \tag{8}$$

$$v_{ri}^t \le \overline{v}_{ri}^t, \qquad \forall r \in R, i \in N: (r,i) \in A\ t \in T \tag{9}$$

5.3 Interconnection lines

An interconnection line is split into two: one for import and one for export. The amount of import and export is bounded by interconnection capacities:

$$0 \le x_{ij}^t \le \overline{x}_{ij}^t, \quad \forall (i,j) \in A^{ICL}, t \in T \tag{10}$$

Ramping constraints on an interconnection line must be satisfied:

$$-\chi_m^t \le \sum_{(i,j) \in A_m^{RAMP+}} \left(x_{ij}^t - x_{ij}^{t-1}\right) - \sum_{(i,j) \in A_m^{RAMP-}} \left(x_{ij}^t - x_{ij}^{t-1}\right) \le \chi_m^t, \qquad \forall m \in RAMP, t \in T\backslash\{0\} \tag{11}$$

$$-\chi_m^t \le \sum_{(i,j) \in A_m^{RAMP+}} \left(x_{ij}^t - x_{ij}^{START}\right) - \sum_{(i,j) \in A_m^{RAMP-}} \left(x_{ij}^t - x_{ij}^{START}\right) \le \chi_m^t, \qquad \forall m \in RAMP, t = 0 \tag{12}$$

5.4 Renewable energy sources

The proposed formulation does not support hydro power or hydro reservoirs. Instead renewable energy stems from sources such as wind and solar energy and is included as an extra area connected to the rest of the system using interconnection lines; see the lower left corner of Figure 1. Existing constraints are thus used to include renewable energy sources:

$$0 \le x_{ij}^t \le \overline{x}_{ij}^t, \quad \forall (i,j) \in A^{ICL}, t \in T \tag{13}$$

where $i \in N$ is the renewable energy area and $j \in N$ the receiving area (e. g. electricity in DK1) and where $\overline{x}_{ij}^t = \infty$. The amount of available renewable energy is set by the inflow variable ψ_i^t and the renewable area balance constraint:

$$\underline{\Psi}_i^t \leq \psi_i^t \leq \overline{\Psi}_i^t, \qquad \forall t \in T \tag{14}$$

$$\psi_i^t \geq 0, \qquad \forall t \in T \tag{15}$$

$$\psi_i^t - \sum_{(i,j)\in A^{ICL}} x_{ij}^t = 0\,, \qquad \forall t \in T \tag{16}$$

The latter constraint is a rewritten version of the balance constraint (2): The remaining variables in the balance constraint are simply not defined for the renewable area.

5.5 Conversion units

The technical minimum and maximum productions must be satisfied:

$$\underline{p}_{hi}^t z_h^t \leq p_{hi}^t \leq \overline{p}_{hi}^t z_h^t, \quad \forall h \in H, i \in N: (h,i) \in A, t \in T \tag{17}$$

A conversion unit can produce up to two different kinds of energy to facilitate Combined Heat and Power (CHP) conversion units. The unit can thus be connected to one or two different areas via the variables p_{hi}^t.

5.5.1 Ramping

Ramping is supported to ensure that production does not increase or decrease too much from hour to hour. If a unit is being turned on, however, it is assumed that it can produce at any production level from the beginning. Similarly, if the unit is turned off, it can also stop production immediately from any production level. The extra functionality at startups and stops is to allow for combinations of low ramping rates and high technical production minima. The ramping constraints are:

$$-\mathrm{P}_{hi}^t - \overline{p}_{hi}^{START}(1 - z_h^t) \leq p_{hi}^t - p_{hi}^{START} \leq \mathrm{P}_{hi}^t + \overline{p}_{hi}^t(1 - z_h^{START}), \qquad \forall h \in H, i \in N: (h,i) \in A, t = 0 \tag{18}$$

$$-\mathrm{P}_{hi}^t - \overline{p}_{hi}^{t-1}(1 - z_h^t) \leq p_{hi}^t - p_{hi}^{t-1} \leq \mathrm{P}_{hi}^t + \overline{p}_{hi}^t(1 - z_h^{t-1})\,, \qquad \forall h \in H, i \in N: (h,i) \in A, t \in T\backslash\{0\} \tag{19}$$

The ramping constraints can be used to tighten the formulation [5] [6]. This is currently not included in SIFRE.

5.5.2 PQ diagram

CHPs can be divided into two subgroups: Backpressure and extraction plants. Backpressure plants can only operate in backpressure mode and thus assumes a fixed relationship between power and heat production. Extraction plants can operate in backpressure mode and as a condensation plant and in all states in between.

First extraction CHPs are considered. The relationship between the two energy types is defined by a PQ diagram[1], which again is defined by the constants c_v and c_b [7]. A PQ diagram example is illustrated in Figure 2. The PQ diagram is based on a fixed modelling of a power plant; in real-life the diagram changes if the operational conditions changes, for example if the fuel mix changes. Using fixed PQ-diagrams are, though, the standard modelling used in the literature [7] [8] [9].

[1] Not to be confused with a synchronous generator P-Q diagram

Define the operating area to be that between the lines with angle c_v, the line with angel c_b and the bounds on power and heat production. The CHP can produce any amount of heat and power within the operating area. The operating area of an extraction CHP is formulated mathematically as follows:

$$p_{hi_1}^t \geq c_b p_{hi_2}^t, \qquad \forall h \in H, i_i, i_2 \in N: (h, i_1), (h, i_2) \in A, t \in T \tag{20}$$

$$p_{hi_1}^t \leq M \cdot z_h^t - c_v p_{hi_2}^t, \qquad \forall h \in H, i_i, i_2 \in N: (h, i_1), (h, i_2) \in A, t \in T \tag{21}$$

$$p_{hi_1}^t \geq m \cdot z_h^t - c_v p_{hi_2}^t, \qquad \forall h \in H, i_i, i_2 \in N: (h, i_1), (h, i_2) \in A, t \in T \tag{22}$$

Backpressure CHPs have a fixed relationship between heat and power production. It is defined by the line with angle c_b and bounded by the technical minima and maxima for heat resp. power production; see the illustration in Figure 2:

$$p_{hi_1}^t = c_b p_{hi_2}^t, \qquad \forall h \in H, i_i, i_2 \in N: (h, i_1), (h, i_2) \in A, t \in T \tag{23}$$

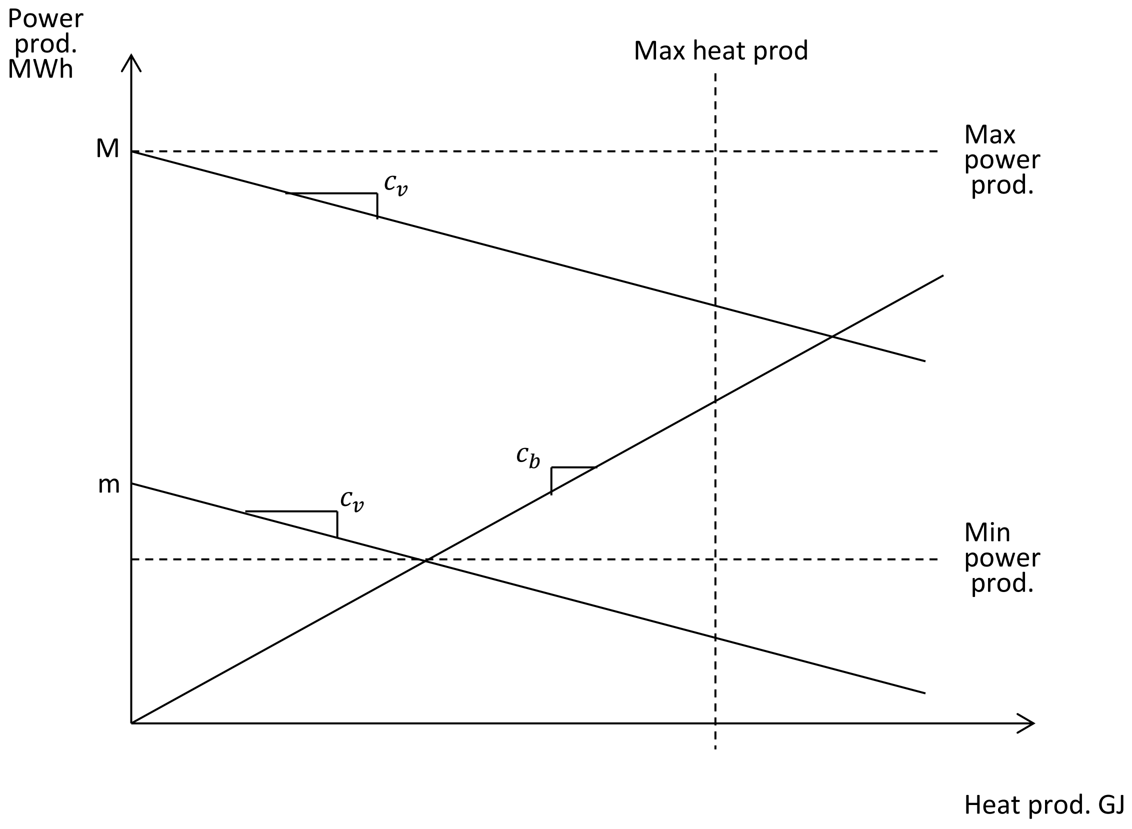

Figure 2 Illustration of a PQ diagram, which defines the relationship between heat and power production

5.5.3 Efficiency

The efficiency of a power plant defines the amount of needed fuel to produce energy and is defined as:

$$f(i_1, i_2, t) = a + b\left(p_{hi_1}^t + p_{hi_2}^t\right) + c\left(\left(p_{hi_1}^t\right)^2 + \left(p_{hi_2}^t\right)^2\right)$$

The efficiency is assumed to be convex, i.e., the fuel consumption is assumed to be non-decreasing when production increases. Efficiency is approximated using piecewise linear functions. The number of pieces depends on the value c. The formulation becomes:

$$\sum_{(i,h)\in A} f_{ih}^t \geq \beta_h^{t\ell} \cdot z_h^t + \alpha_h^{t\ell}(p_{hi_1}^t + p_{hi_2}^t) \;\; \forall \ell \in pieces(h,t), h \in H, i_i, i_2 \in N: (h, i_1), (h, i_2) \in A, t \in T \qquad (24)$$

To take into account the possibility of negative fuel prices, it is not sufficient to set a minimum bound of fuel usage. A tight upper bound on the convex efficiency functions requires the introduction of binary variables. The constant c is in practice[2] assumed to be very small, hence an upper bound can be set by drawing a line between the two endpoints of the efficiency constraints:

$$f_{max}^t = \bar{\beta}_h^t + \bar{\alpha}_h^t p_{max}^t$$

$$f_{min}^t = \bar{\beta}_h^t + \bar{\alpha}_h^t p_{min}^t$$

The maximal and minimum production amounts are derived using the PQ diagram. The upper bound constraint is derived:

$$\bar{\alpha}_h^t = \frac{f_{max}^t - f_{min}^t}{p_{max}^t - p_{min}^t}$$

$$\bar{\beta}_h^t = f_{max}^t - \bar{\alpha}_h^t p_{max}^t$$

$$\sum_{(i,h)\in A} f_{ih}^t \leq \bar{\beta}_h^t \cdot z_h^t + \bar{\alpha}_h^t(p_{hi_1}^t + p_{hi_2}^t) \;\; \forall h \in H, i_i, i_2 \in N: (h, i_1), (h, i_2) \in A, t \in T \qquad (25)$$

The efficiency constants may depend on the production range. For example, the efficiency may be slightly worse in the upper production ranges, because the conversion unit is designed to generate energy in its mid-interval. The different efficiencies are concatenated. The following algorithm ensures a convex overall efficiency which can be linearized:

1. Calculate fuel consumption at a number of production samples
2. Calculate the line between neighboring samples
3. If an angle of any line is smaller than the angle of the next line
    - Delete one of the end points of the line
    - Go to step 2

The quality of this approximation of fuel consumption is theoretically poor. However, in practice it is fair to assume that the efficiencies are already convex (or close to convex).

Changing the efficiency constants a, b, c is often used to simulate that a conversion unit supports overproduction: Some production units are capable of exceeding their official technical production maximum for a brief period of time. This functionality should not be used unless needed, because of extra

[2] For the Danish power and heat system. Also for the benchmarks instances [42] often used in the literature [26] [27].

wear and tear. To make it very unattractive to enter the overproduction state, the efficiency for the production interval causes a very large fuel usage. Furthermore, the technical production maximum of the unit must be increased to include over production.

5.5.4 Distribution of fuel usage

A conversion unit may use a mix of fuels, e.g. up to 80% coal and 50% oil. The sum of maximum bounds on fuel usage must be 100%. The total fuel consumption is defined by the efficiency constraints and the restriction on fuel usage is ensured by constraints:

$$f_{ih}^{t} \leq m_{ih} \sum_{(j,h)\in A} f_{jh}^{t}, \quad \forall h \in H, i \in N: (i,h) \in A, t \in T \tag{26}$$

5.5.5 Startup consumption

The fuel consumption of turning on a conversion unit depends on how long, it has been offline:

$$f(t) = \ddot{a} + \ddot{b}\left(1 - e^{-\frac{t}{\ddot{T}}}\right)$$

Given some fuel cost, the startup cost of a conversion unit is illustrated in Figure 3. As seen in the Figure, the startup cost increases with the offline time. This means that the offline time should be bounded from below.

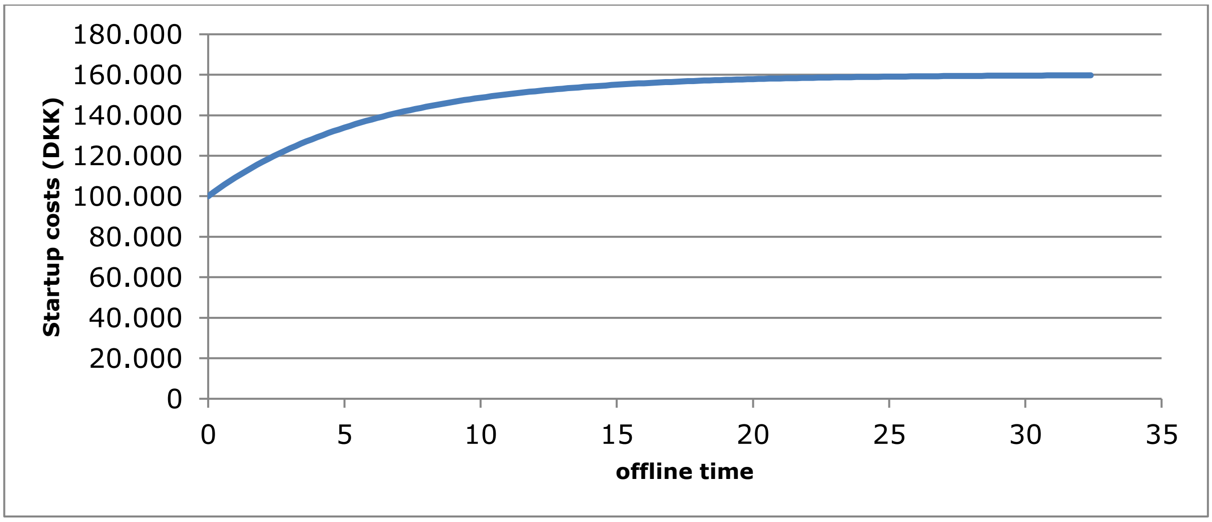

Figure 3 Illustration of the startup cost of a conversion unit

Let L be a set of time steps between 0 and l, where the latter is some threshold value where the startup consumption curve flattens. The offline time of a conversion unit is derived:

$$z_{h}^{t} - \sum_{t'=t-\ell}^{t-1} z_{h}^{t'} - \sum_{\ell'=\ell+1}^{|L|} y_{h}^{t\ell'} \leq y_{h}^{t\ell}, \quad \forall h \in H, t \in T, \ell \in L \tag{27}$$

The constraints work when the startup costs are negative. Fuel costs can, however, be negative. In this case an upper bound on the $y_{h}^{t\ell}$ is necessary:

$$\sum_{\ell \in L} y_h^{t\ell} \leq z_h^t, \qquad \forall t \in T, h \in H$$

$$y_h^{t\ell} \leq 1 - z_h^{t'}, \qquad \forall t \in T, h \in H, \ell \in L, t' \in \{t-\ell, \dots, t-1\}$$

First constraint ensures that at most one $y_h^{t\ell}$ is set to one. The second constraint ensures that the activated $y_h^{t\ell}$ is not "too early". The upper bounds hurt performance, as they eliminate the possibility of Gurobi LP-relaxing the $y_h^{t\ell}$ variables. They are hence only added when necessary (when the startup costs can be negative).

5.5.5.1 Simplified startup costs

The objective function includes fuel consumption for startups. The startups can instead be formulated simply as a cost, without taking the actual fuel consumptions into account. This is a simplified approach, as the cost would have to be calculated up front even if the fuel consumption costs are not fully known. The UC with the simplified approach, however, is faster to solve. The term $\mathrm{f}_{\mathrm{ih}}^{\mathrm{ts}}$ must be removed from the objective function and instead the term must be added:

$$\sum_{h \in H} \sum_{t \in T} \sum_{\ell \in L} c_h^{t\ell} \, y_h^{t\ell}$$

where $c_h^{t\ell}$ is the pre-calculated startup cost, i.e. $c_h^{t\ell} = \ddot{a} + \ddot{b}\left(1 - e^{-\frac{t}{\ddot{T}}}\right)$

5.5.5.2 Optimized startup fuel consumption

The current objective function assumes that startup fuel consumptions are optimized. This requires an extra constraint, which upper bounds the amount of fuel, which can be used as startup consumption:

$$f_{ih}^{ts} \leq m_{ih}^{s} \sum_{\ell \in L} \overline{f_h^{ts}} \cdot y_h^{t\ell}, \quad \forall t \in T, i \in N, h \in H$$

Where $\overline{f_h^{ts}} = \ddot{b}\left(1 - e^{-\frac{t}{\ddot{T}}}\right)$ and m_{ih}^{s} is the percentage of fuel $i \in N$, which can be used as startup fuel by conversion unit $h \in H$. Another constraint is needed to ensure that sufficient fuel is used for startup:

$$\sum_{(i,h) \in A} f_{ih}^{ts} = \sum_{\ell \in L} \overline{f_h^{ts}} \cdot y_h^{t\ell}, \quad \forall t \in T, h \in H$$

5.5.6 Maintenance

Recall that in layer 1, SIFRE simulates a full year with low detail in order to decide when conversion units should be taken out for maintenance. The constraints for including maintenance are:

$$1 - z_h^{t'} \geq zm_h^t, \qquad \forall\, t \in T, t' \in \{t, \dots, t + m_h\}, h \in H \tag{28}$$

$$\sum_{t=0}^{|T|} zm_h^t = M_h, \qquad \forall h \in H \tag{29}$$

$$\sum_{t'=t}^{t+m_h+\underline{m}_h} zm_h^{t'} \leq 1, \qquad \forall t \in T, h \in H \tag{30}$$

$$zm_h^t - \sum_{t'=t+m_h}^{t+m_h+\bar{m}_h} zm_h^{t'} \leq 0, \qquad \forall t \in T, h \in H \tag{31}$$

The first constraints ensure that the unit can only be taken out for maintenance when offline and it is not turned on when maintained. The second constraint makes sure that the unit is taken out to revision the correct number of times. The final two constraints force maintenance within the time bounds: two neighboring maintenance periods should at least be $\underline{m}_h$ and most be $\bar{m}_h$ time apart.

The variables zm_h^t cannot be LP-relaxed without losing precision. Considering the low level of detail in layer 1, the number of variables, however, should not be too large compared to the problem instance size.

5.5.7 Outages

The term "outages" is used for unplanned events at conversion units. Outages can be given as input by adjusting the installed capacity, or they can be stochastically generated by the UC model. The latter case only generates outages, which result in zero capacity. Outages are sampled using values for the average outage length (l_{out}^h) and for the percentage of time in spent outage (p_{out}^h) for a unit $h \in H$. Outages are generated stochastically before solving each simulation period in layer 2. Perfect foresight is assumed, but outages are not taken into account before the next day: The intraday imbalance caused by an outage must be handled in the intraday market, not by the spot market (i.e. by SIFRE).

The time not in outage is generated stochastically using a uniform distribution with mean set to the average time not in outage. The length of an outage is sampled using the exponential distribution with mean set to the average outage length. Sampling outages thus corresponds to sampling the waiting time until the unit can produce energy again.

5.6 Electric vehicles

Electric vehicles are represented on aggregated form, rather than as individual components. Electric vehicles are included using existing constraints: A set of electric vehicles is considered as an extra electricity demand to be satisfied in an area. The formulation supports the vehicle-to-grid technology, such that electric vehicles can be used as batteries. Figure 4 illustrates how electric vehicles are included.

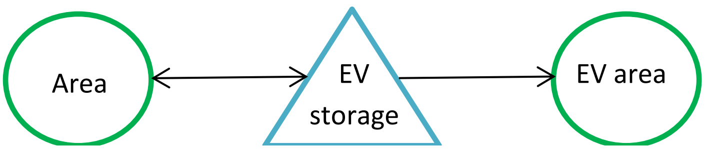

Figure 4 Illustration of how electric vehicles are represented

The constraints correspond to those for storages. Let $i \in N$ be the area, to which the electric vehicles are connected (the leftmost area in Figure 4), $e \in E$ be the area representing electric vehicles (the rightmost area in Figure 4) and $r \in R_E$ be the electric vehicle storages (the storage in Figure 4). The constraints are:

$$s_r^t = (1-\gamma_r^t)s_r^{START} + \left((1-\gamma_{ir}^t)v_{ir}^t - v_{ri}^t\right) - v_{re}^t, \quad \forall r \in R_e, e \in E: (r,e) \in A, t = 0 \tag{32}$$

$$s_r^t = (1-\gamma_r^t)s_r^{t-1} + \left((1-\gamma_{ir}^t)v_{ir}^t - v_{ri}^t\right) - v_{re}^t, \qquad \forall r \in R_E, e \in E{:}\,(r,e) \in A, t \in T\backslash\{0\} \tag{33}$$

$$\underline{s}_r^t \le s_r^t \le \overline{s}_r^t, \qquad \forall r \in R_E, t \in T \tag{34}$$

As for the storage constraints (5)-(6), these constraints ensure that the storage level is updated correctly according to injection, extraction and losses. The demand in the electric vehicle area $j \in N$ represents the energy consumption of the electric vehicles and must be satisfied:

$$v_{re}^t = d_e^t, \qquad \forall r \in R_E, e \in E{:}\,(r,e) \in A, t \in T \tag{35}$$

In any given time step, the storage level must be large enough for the demand in the next time step to be satisfied to prevent the need for extension cords:

$$s_r^t \ge d_e^{t+1}, \qquad \forall r \in R_E, e \in E{:}\,(r,e) \in A, t \in T \tag{36}$$

The injection and extraction rates are limited by the number of vehicles plugged into a charger. This again depends on the consumption profile: If the electric vehicle consumption is high in a time step, then relatively many vehicles are using electricity and their batteries are not available to the power system. Let K_e be the percentage of vehicles unavailable to the power system, when the electric vehicle consumption is at its highest. Then:

$$K_e \cdot fleet_e = \max_{t \in T} d_e^t \Leftrightarrow fleet_e = \frac{\max\limits_{t \in T} d_e^t}{K_e}$$

The percentage of vehicles unavailable at any given time step is then:

$$\frac{d_e^t}{fleet_e}$$

The injection and extraction rates are:

$$v_{ri}^t \le \overline{v}_{ri}^t \cdot \left(1 - \frac{d_e^t}{fleet_e}\right), \qquad \forall r \in R_E, e \in E{:}\,(r,e) \in A, i \in N{:}\,(i,r) \in A, t \in T \tag{37}$$

$$v_{ir}^t \le \overline{v}_{ir}^t \cdot \left(1 - \frac{d_e^t}{fleet_e}\right), \qquad \forall r \in R_E, e \in E{:}\,(r,e) \in A,, i \in N{:}\,(i,r) \in A, t \in T \tag{38}$$

Injection and extraction are limited according to available vehicles to represent how much energy is available to the power system. When vehicles leave the charger, they can carry more energy with them than necessary. Limiting the battery capacity according to available vehicles would eliminate this possibility.

6. Final mathematical formulation

For the sake of overview, the constraints and objective function are gathered to the final mathematical formulation. For explanation of constraints, see the previous sections.

$$\min Z = \sum_{t\in T}\left(\sum_{i\in N}\left(\widehat{\psi}_i^t\psi_i^t + \sum_{l\in L(i)}(co_i^l o_i^{lt} + cu_i^l u_i^{lt}) + \sum_{(i,h)\in A} c_{ih}^t(f_{ih}^t + f_{ih}^{ts}) + \sum_{(h,i)\in A}\ddot{c}_{hi}^t p_{hi}^t\right)\right.$$
$$\left. + \sum_{(i,j)\in A^{ICL}}\bar{c}_{ij}^t x_{ij}^t + \sum_{h\in H}\tilde{c}_h^t z_h^t\right) - \sum_{r\in R}\hat{\rho}_r^{END} s_r^{t'} - \sum_{t\in T}\sum_{i\in N}\sum_{\ell\in D(i)}\pi_\ell^t\,\widetilde{d}_\ell^t$$
$$+\sum_{i\in N}\sum_{(j,t)\in F(i)}\sum_{\tau=t-k_j}^{t+k_j}\bar{\bar{c}}_{jt}^{\tau}\,\ddot{d}_j^{t\tau} \tag{39}$$

$$\psi_i^t + \sum_{(h,i)\in A} p_{hi}^t - \sum_{(i,h)\in A}(f_{ih}^t + f_{ih}^{ts}) + \sum_{(j,i)\in A^{ICL}} x_{ji}^t - \sum_{(i,j)\in A^{ICL}}\gamma_{ij}^t x_{ij}^t - \sum_{(r,i)\in A}(v_{ir}^t - \gamma_{ri}^t v_{ri}^t) - o_i^t + u_i^t$$
$$= d_i^t + \sum_{\ell\in D(i)}\tilde{d}_{i\ell}^t + \sum_{t'\in F(i)}\ddot{d}_j^{t't}, \qquad \forall i\in N, t\in T \tag{40}$$

$$\underline{\Psi}_i^t \le \psi_i^t \le \overline{\Psi}_i^t, \qquad \forall i\in N, t\in T \tag{41}$$

$$\sum_{\tau=t-k_j}^{t+k_j}\ddot{d}_j^{t\tau} = \overline{d_j^t}, \qquad \forall i\in N, (j,t)\in F(i) \tag{42} \quad (43)$$

$$s_r^t = (1-\gamma_r^t)s_r^{START} + \sum_{\substack{i\in N:\\(i,r),(r,i)\in A}}((1-\gamma_{ir}^t)v_{ir}^t - v_{ri}^t), \quad \forall r\in R, t=0 \tag{44}$$

$$s_r^t = (1-\gamma_r^t)s_r^{t-1} + \sum_{\substack{i\in N\\(i,r),(r,i)\in A}}((1-\gamma_{ir}^t)v_{ir}^t - v_{ri}^t), \qquad \forall r\in R, t\in T\backslash\{0\} \tag{45}$$

$$\underline{s}_r^t \le s_r^t \le \overline{s}_r^t, \qquad \forall r\in R, t\in T \tag{46}$$

$$v_{ir}^t \le \underline{v}_{ir}^t, \qquad \forall r\in R, i\in N: (i,r)\in A, t\in T \tag{47}$$

$$v_{ri}^t \le \overline{v}_{ri}^t, \qquad \forall r\in R, i\in N: (r,i)\in A\ t\in T \tag{48}$$

$$0 \le x_{ij}^t \le \overline{x}_{ij}^t, \quad \forall (i,j)\in A^{ICL}, t\in T \tag{49}$$

$$-\chi_m^t \le \sum_{(i,j)\in A_m^{RAMP+}}\left(x_{ij}^t - x_{ij}^{t-1}\right) - \sum_{(i,j)\in A_m^{RAMP-}}\left(x_{ij}^t - x_{ij}^{t-1}\right) \le \chi_m^t, \qquad \forall m\in RAMP, t\in T\backslash\{0\} \tag{50}$$

$$-\chi_m^t \le \sum_{(i,j)\in A_m^{RAMP+}}\left(x_{ij}^t - x_{ij}^{START}\right) - \sum_{(i,j)\in A_m^{RAMP-}}\left(x_{ij}^t - x_{ij}^{START}\right) \le \chi_m^t, \qquad \forall m\in RAMP, t=0 \tag{51}$$

$$\underline{p}_{hi}^t z_h^t \le p_{hi}^t \le \overline{p}_{hi}^t z_h^t, \quad \forall h\in H, i\in N: (h,i)\in A, t\in T \tag{52}$$

$$-\mathrm{P}_{hi}^t - \overline{p}_{hi}^{START}(1-z_h^t) \le p_{hi}^t - p_{hi}^{START} \le \mathrm{P}_{hi}^t + \overline{p}_{hi}^t(1-z_h^{START}),$$
$$\forall h\in H, i\in N: (h,i)\in A, t=0 \tag{53}$$

$$-\mathrm{P}_{hi}^t - \overline{p}_{hi}^{t-1}(1-z_h^t) \le p_{hi}^t - p_{hi}^{t-1} \le \mathrm{P}_{hi}^t + \overline{p}_{hi}^t(1-z_h^{t-1}), \qquad \forall h\in H, i\in N: (h,i)\in A, t\in T\backslash\{0\} \tag{54}$$

$$p_{hi_1}^t \ge c_b p_{hi_2}^t, \qquad \forall h\in H(condensation\ CHP),, i_i, i_2\in N: (h,i_1),(h,i_2)\in A, t\in T \tag{55}$$

$$p_{hi_1}^t \le M\cdot z_h^t - c_v p_{hi_2}^t, \qquad \forall h\in H(condensation\ CHP),, i_i, i_2\in N: (h,i_1),(h,i_2)\in A, t\in T \tag{56}$$

$$p_{hi_1}^t \geq m \cdot z_h^t - c_v p_{hi_2}^t, \qquad \forall h \in H(condensation\ CHP), i_i, i_2 \in N: (h, i_1), (h, i_2) \in A, t \in T \tag{57}$$

$$p_{hi_1}^t = c_b p_{hi_2}^t, \qquad \forall h \in H(extraction\ CHP), i_i, i_2 \in N: (h, i_1), (h, i_2) \in A, t \in T \tag{58}$$

$$\sum_{(i,h) \in A} f_{ih}^t \geq \beta_h^{t\ell} \cdot z_h^t + \alpha_h^{t\ell}(p_{hi_1}^t + p_{hi_2}^t) \ \ \forall \ell \in pieces(h,t), h \in H, i_i, i_2 \in N: (h, i_1), (h, i_2) \in A, t \in T \tag{59}$$

$$\sum_{(i,h) \in A} f_{ih}^t \leq \bar{\beta}_h^t \cdot z_h^t + \bar{\alpha}_h^t(p_{hi_1}^t + p_{hi_2}^t) \ \ \forall h \in H, i_i, i_2 \in N: (h, i_1), (h, i_2) \in A, t \in T \tag{60}$$

$$f_{ih}^t \leq m_{ih} f_h^t, \quad \forall h \in H, i \in N: (i, h) \in A, t \in T \tag{61}$$

$$z_h^t - \sum_{t'=t-\ell}^{t-1} z_h^{t'} - \sum_{\ell'=\ell+1}^{|L|} y_h^{t\ell'} \leq y_h^{t\ell}, \quad \forall h \in H, t \in T, \ell \in L \tag{62}$$

$$\sum_{\ell \in L} y_h^{t\ell} \leq z_h^t, \qquad \forall t \in T, h \in H \tag{63}$$

$$y_h^{t\ell} \leq 1 - z_h^{t'}, \qquad \forall t \in T, h \in H, \ell \in L, t' \in \{t - \ell, \dots, t-1\} \tag{64}$$

$$f_{ih}^{ts} \leq m_{ih}^s \sum_{\ell \in L} \overline{f_h^{ts}} \cdot y_h^{t\ell}, \quad \forall t \in T, i \in N, h \in H \tag{65}$$

$$\sum_{(i,h) \in A} f_{ih}^{ts} = \sum_{\ell \in L} \overline{f_h^{ts}} \cdot y_h^{t\ell}, \quad \forall t \in T, h \in H \tag{66}$$

$$s_r^t = (1 - \gamma_r^t) s_r^{START} + \left((1 - \gamma_{ir}^t) v_{ir}^t - v_{ri}^t\right) - v_{re}^t, \quad \forall r \in R_e, e \in E: (r, e) \in A, t = 0 \tag{67}$$

$$s_r^t = (1 - \gamma_r^t) s_r^{t-1} + \left((1 - \gamma_{ir}^t) v_{ir}^t - v_{ri}^t\right) - v_{re}^t, \qquad \forall r \in R_E, e \in E: (r, e) \in A, t \in T \backslash \{0\} \tag{68}$$

$$\underline{s}_r^t \leq s_r^t \leq \overline{s}_r^t, \qquad \forall r \in R_E, t \in T \tag{69}$$

$$v_{re}^t = d_e^t, \qquad \forall r \in R_E, e \in E: (r, e) \in A, t \in T \tag{70}$$

$$s_r^t \geq d_e^{t+1}, \qquad \forall r \in R_E, e \in E: (r, e) \in A, t \in T \tag{71}$$

$$v_{ri}^t \leq \overline{v}_{ri}^t \cdot \left(1 - \frac{d_e^t}{fleet_e}\right), \qquad \forall r \in R_E, e \in E: (r, e) \in A, i \in N: (i, r) \in A, t \in T \tag{72}$$

$$v_{ir}^t \leq \overline{v}_{ir}^t \cdot \left(1 - \frac{d_e^t}{fleet_e}\right), \qquad \forall r \in R_E, e \in E: (r, e) \in A, , i \in N: (i, r) \in A, t \in T \tag{73}$$

Maintenance is only included in the first layer:

$$1 - z_h^{t'} \geq zm_h^t, \qquad \forall\ t \in T, t' \in \{t, \dots, t + m_h\}, h \in H \tag{74}$$

$$\sum_{t=0}^{|T|} zm_h^t = M_h, \qquad \forall h \in H \tag{75}$$

$$\sum_{t'=t}^{t+m_h+\underline{m}_h} zm_h^{t'} \leq 1, \qquad \forall t \in T, h \in H \tag{76}$$

$$zm_h^t - \sum_{t'=t+m_h}^{t+m_h+\overline{m}_h} zm_h^{t'} \leq 0, \qquad \forall t \in T, h \in H \tag{77}$$

7. Resulting area prices

An energy price must be derived for each area. For electricity areas, this corresponds to finding the market clearing price from the spot market.

7.1 Obtaining a dual solution

The energy price is set to the dual of the balance constraint (47). The dual variable values are derived by first solving the MILP to optimality, then fix the values of integer variables and then resolve the problem as an LP. This is a time-consuming process; alternatively, a separate algorithm could be developed to calculate the area prices using the integer solution from the MILP.

An analysis is performed to better understand the values of the dual variable of the balance constraint (47) and to compare it with the expected spot market price.

7.2 Marginal costs

The dual variable of balance constraint (47) reflects the objective function cost/profit of increasing the right-hand side. The balance constraint only considers costs per MWh energy and not costs such as operation cost (per hour) or startup costs (per startup). The area price is thus the true marginal production cost, ignoring operation and startup costs.

A conversion unit would in the real world not generate energy, unless its expenses are covered. The operation cost and startup costs can be included in the area prices via post processing. In case of CHPs, it must be decided if the costs should be covered by the power or heat area price or a combination.

In SIFRE the extra costs are covered by the heat area price when possible, because in Denmark the heat prices are determined bilaterally and not by a market clearing. The extra costs are simply added to the current production costs of each conversion units, such that new marginal costs can be derived. This method is not perfect. In the spot market, a conversion unit may submit very cheap bids to avoid being turned off. This behavior is captured by the dual variable values of constraint (47) but eliminated again, if the marginal costs are altered.

8. Computational results

SIFRE is evaluated with respect to quality and running time. The former is done by simulating the Danish power system on historical data for 2013 and compare the results with actual the spot market prices. The running time is measured by using the UC model to solve benchmarks from the literature. The proposed branching strategy is tested to see if performance is improved.

8.1 Quality assessment

SIFRE is run on historical data from 2013 for Denmark with price areas and interconnection lines as illustrated in Figure 5. Most data is publicly available and includes the following:

- Capacities on interconnection lines [10]
- Ramping constraint on interconnection lines [10]
- Hourly power prices from the neighboring areas: NO, SE3, SE4 and DE [10]
- Realized power consumption in DK1 and DK2 [11]

- Estimated heat consumption in Denmark [12] (not all details are publicly available)
- Representation of the conversion units in DK1 and DK2 [12] (not all details are publicly available)
- Maintenance schedules [13]
- Day ahead wind prognoses (not publicly available)

Outages (that is unplanned contingencies on conversion units) are generated stochastically. Maintenance schedules are only available for generation units with more than 100 MW installed capacity.

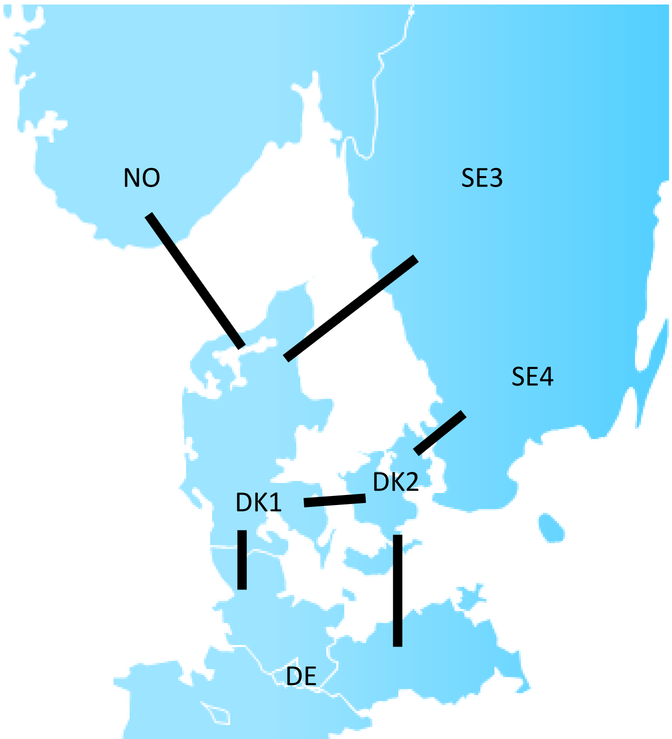

Figure 5 Illustration of the Danish power system with interconnection lines to neighbor price areas

In the spot market, it is generally desirable to deliver energy from SE3 directly to DK1, rather than via SE4 and DK2. This is especially a relevant observation during spring and early summer where hydro power is cheap and flows from Northern Sweden to the South. To simulate this behavior, a small net tariff is added to the interconnection line between DK2 and SE4.

Results are compared to area prices and interconnection line flows from the spot market [10]. Market clearing failed in a number of hours on June 7 2013, due to imperfect behavior of market participants. SIFRE is unable to simulate this behavior, because we could not derive the data to represent the event. Thus the hours from 7:00 – 12:00 am on June 7, 2013 are left out of the comparison.

SIFRE calculates the average prices for DK1 and DK2 very well as seen in Table 1.

Average prices	Nord Pool Spot prices excl. even on June 7, 2013	SIFRE
DK1	282,60 DKK	282,74 DKK
DK2	295,37 DKK	296,29 DKK

Table 1 Average prices: Nord pool spot vs SIFRE simulation

A more detailed comparison confirms the accuracy of the SIFRE simulation. Consider the histogram in Figure 6: SIFRE calculates the right price in more than 7000 hours and 8000 hours for DK1 resp. DK2: The yearly number of hours is 8760. Only few hours in SIFRE deviates significantly from the spot market prices.

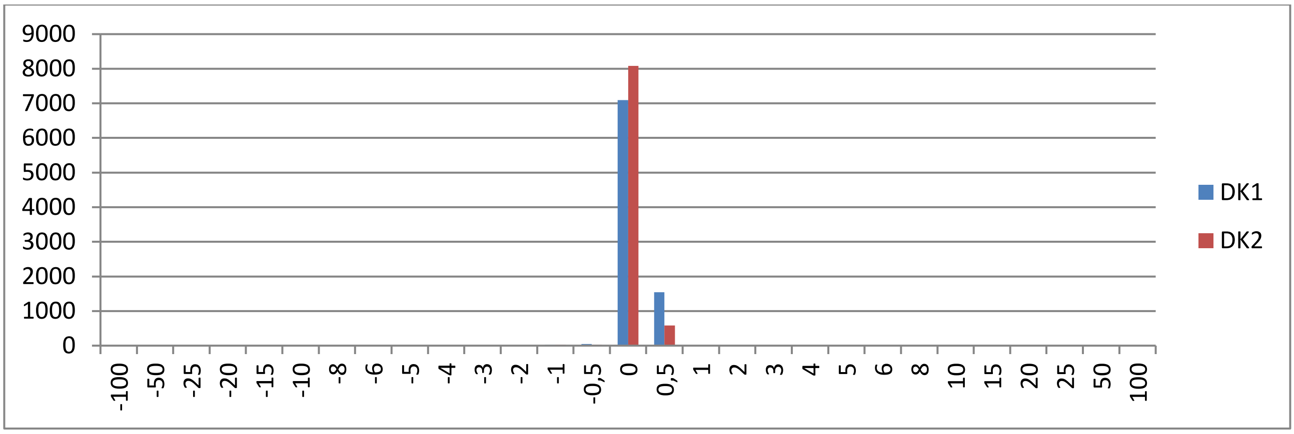

Figure 6 Histogram of the hourly deviation between SIFRE and spot market prices for DK1 and DK2, measured in percent. The y-axis counts the number of hours. The total number of hours in the year is 8760.

To reduce the length of this Section, only selected interconnection line flows are presented. Flows on the remaining interconnection lines behave similarly. Examples of flows are seen in Figure 7, Figure 8 and Figure 9, where the monthly sums are given. SIFRE is capable of simulating the real-life behavior well, when considering the monthly results. The hourly results are less accurate, because of differences in unplanned outages. The Storebælt interconnection line between DK2 and DK1 is considered the most difficult one to simulate correctly, because none of its end areas (DK2 and DK1) are static. SIFRE simulates the results on this line well.

The deviation in prices and in flows can first of all be explained by the inaccuracy in input data. Especially unplanned outages may be to blame. Furthermore, the test is performed on data where Sweden and Norway are assumed to be static (e.g. they have a fixed price). In reality, the spot market generates market clearings for the entire Nordic Market at the same time; this is not represented in the data instance.

Based on backtest experiences at Energinet.dk, the results of SIFRE are very satisfying. The results do not indicate that the UC formulation is in any way incorrect. Instead, the results emphasize the importance of accurate data when performing a backtest.

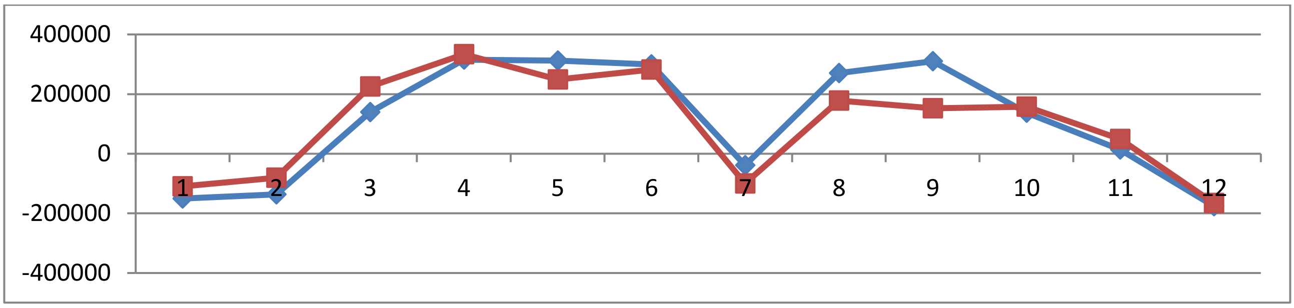

Figure 7 Illustration of monthly flow sums for the interconnection line between DK1 and DE. Positive values indicate import to DK1 and negative values export to DE. The y-axis represents MWh and the x-axis months. The blue line represents the spot market and the right line SIFRE

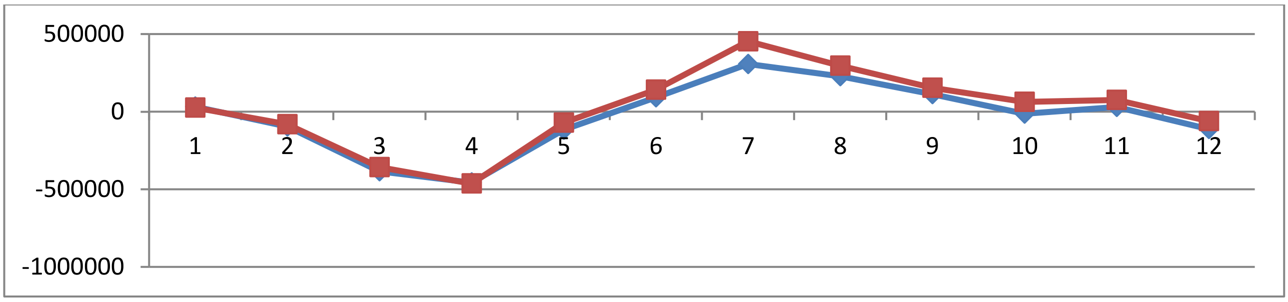

Figure 8 Illustration of monthly flow sums for the interconnection line between DK1 and NO. Positive values indicate import to DK1 and negative values export to NO. The y-axis represents MWh and the x-axis months. The blue line represents the spot market and the right line SIFRE

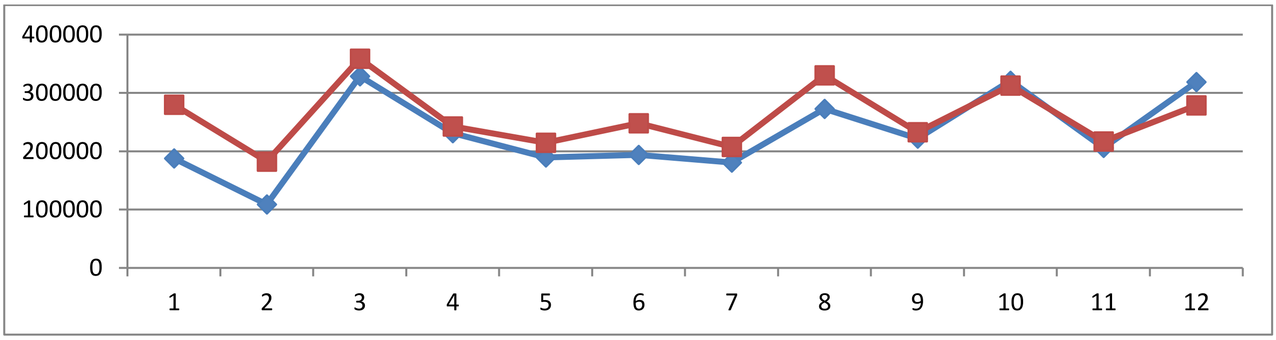

Figure 9 Illustration of monthly flow sums for the interconnection line between DK2 and DK1. Positive values indicate import to DK2 and negative values export to DK1. The y-axis represents MWh and the x-axis months. The blue line represents the spot market and the right line SIFRE

9. Concluding remarks

This paper introduced the energy spot market simulation tool: SIFRE - Simulating Flexible and Renewable Energy sources. SIFRE complements the many existing simulation tools with its generic and flexible representation of energy systems. The complete mathematical formulation behind SIFRE is presented; the formulation is based on the Unit Commitment problem, but with a much greater level of detail when it comes to energy production and fuel consumption.

The quality of SIFRE is assessed using historic data for the Danish power and heating system in 2013. The test results reveal that SIFRE is capable of simulating well the real-life behavior of the Nordpool spot market. Inaccuracies in the results are most likely due to inaccuracies in the input data. The simulation time of SIFRE is also assessed through a detailed parametric study, focusing on the convergence towards an integer solution. The computational study reveals that it is worthwhile considering more sophisticated method to help the MILP solver – in this case Gurobi – converge.

Currently, SIFRE is iteratively being taken into use at Energinet.dk, the Danish Transmission System Operator. A large part of the development work has been the user experience, which together with high quality results is crucial for success. SIFRE is currently also being integrated into research projects such as “CITIES – Centre for IT-Intelligent Systems in Cities” [14] and “Symbio – Biogasupgrade” [15]. The development of SIFRE continues, both with respect to functionality and usability.

9.1 Future work

SIFRE can be extended with plenty of functionality. The most important is probably hydropower, which is often necessary when modelling energy systems. Currently, hydro power can be included in SIFRE as an externally given component with known storage levels and prices. A fuller inclusion of hydro power would include an algorithm to decide the storage levels and energy prices.

SIFRE is a deterministic model with respect to renewable energy. SIFRE could be extended to include a stochastic representation of e.g. wind. Stochasticity may produce results of better quality, because several wind scenarios can be taken into account. The simulation time, however, would suffer greatly. An example of this is Wilmar [16].

10. References

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